# CN510 Assignment 2: Numerical Integration of Leaky Integrator and Leaky Integrate-and-Fire Neuron

Due Tuesday Sept. 24, 2013

## Leaky Integrator

Continue investigation of a leaky integrator.

$$\frac{dx}{dt} = -Ax + I \tag{1}$$

### Hand-made Euler Numerical Integration

In Matlab (or your choice of programming language) code up a function for the right hand side of equation (1). The Euler forward integration method can be described as

$$\frac{\Delta x}{\Delta t} = f_{RHS}(x) \tag{2}$$

or

$$x(n) = x(n-1) + \Delta x = x(n-1) + f_{RHS}(x(n-1))\Delta t \tag{3}$$

So you can use the same iterative approach you have used in the previous assignment for Rotter-Diessmann part, except here you will be computing $\Delta x$ and adding it to $x$ rather than $x$ directly as you did last time. Select a reasonable integration time step $\Delta t$ so that input manipulations below produce smooth solution trajectories. Run the simulation for $T = 10$ time units and turn the input off at $t = 5$. This way you will receive four cases that you have used in the last assignment:

- $I = 5$, $x_0 = 0$, $A = 1$;
- $I = 0$, $x_0 =$ whatever solution of the previous case gives you at $t = 5$, $A = 1$;
- $I = 5$, $x_0 = 0$, $A = 2$;
- $I = 0$, $x_0 =$ whatever solution of the previous case gives you at $t = 5$, $A = 2$;

This time though, cases 1 and 2 will be combined in simulation 1 and cases 3 and 4 will be combined in simulation 2. Take the value of $x$ at times $t = 5$ and $t = 10$ and compare them to the results of the analytical solution and Rotter-Diessmann solution from the previous assignment. Plot the results of your simulations through time (label the axes, include parameters in the captions). How do they look in comparison to previous assignment results?

Anticipating a possible concern from advanced MatLab users: yes, it is possible and somewhat simpler to use built-in MatLab numerical integration methods for this assignment. Unfortunately, making these methods to handle spiking output and especially spiking input is non-trivial, so some practice with coding your own solvers will become handy later in the course.

# Leaky Integrate and Fire

Add the following conditional statement to the equation (1) above to create leaky integrate and fire neuron:

$$x = \begin{cases} x, & \text{if } x < \theta \\ -1, spike, & \text{if } x \geq \theta \end{cases} \tag{4}$$

## Hand-made Euler Numerical Integration

Modify your code from the first part to include the after-spike reset (equation 4). Note that here you only have two operations: update and reset, so it does not matter much (only during the first and last step) in which order you will perform these operations in your loop. Run the simulation for $T = 10$ time units and turn the input $I$ on at $t = 1$ and off at $t = 6$. During the on time set $I = 3$, during off time $I = 0$. Use two values of the spiking threshold $\theta = 1$ and $\theta = 2$. Set decay rate $A = 1$.

Your report shall include answers to the following tasks.

- Plot the spike events through time with labeled axes and be sure to include the parameter values in the legend.
- Calculate the spike rates for two different thresholds: take the number of spikes the model produced and divide by time interval between first and last spike. What is the ratio of spike rates for two different thresholds?
- What is the ratio between times to first spike (from the moment input is turned on) for two threshold values?

## Additional Point to Discuss in the Report

- Include your code in the appendix. Highlight the differences between integrator and integrate-and-fire.
- Hypothesize how a change of $A$ from 1 to 2 will affect spike rates and delay till first spike in the leaky Integrate-and-Fire based on your results for leaky integrator. Justify your prediction.

## Grading Rubric:

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|---|---|
| 45 points | **Leaky integrator** from which |
| 25 points | Well-formatted plot with readable labels and parameter meanings listed in the caption |
| 20 points | Answers to all the questions above |
| 45 points | **Leaky integrator** from which |
| 25 points | Well-formatted plot with readable labels and parameter meanings listed in the caption |
| 20 points | Answers to all the questions above |
| 10 points | **Prediction of the effect of decay rate on lIaF dynamics** |