

# CN510 Assignment 7: The Outstar Network

Due Thursday Oct. 31, 2013

The purpose of this assignment is to simulate an outstar network with three border cells. The equations for such a network can be summarized as follows:

$$\frac{dx_0}{dt} = -A_0x_0 + I_0 \quad (1)$$

$$\frac{dx_i}{dt} = -Ax_i + B[x_0(t - \tau) - \Gamma]_+w_{0i} + I_i \quad (2)$$

$$\frac{dw_{0i}}{dt} = -Cw_{0i} + D[x_0(t - \tau) - \Gamma]_+x_i \quad (3)$$

where  $x_0$  is the activity of the source cell, the  $x_i$  are the activities of the border cells ( $i=1,2,3$ ), and the  $w_{0i}$  are synaptic weights. The constants  $A_0, A, B, C, D, \Gamma$ , and  $\tau$  are network parameters restricted to non-negative values. The signal  $I_0$  can be thought of as the conditioning stimulus, and the signals  $I_i$  can be thought of as the unconditioned stimuli. The source cell sampling signal  $S_0(t) = [x_0(t - \tau) - \Gamma]_+$  is defined with the threshold linear function  $[a]_+ = \max(a, 0)$  and is delayed by  $\tau$  seconds before reaching the border cells. Pattern variables can be defined as:

$$\Theta_i = \frac{I_i}{\sum_k^3 I_k} \quad (4)$$

$$X_i = \frac{x_i}{\sum_k^3 x_k} \quad (5)$$

$$W_{0i} = \frac{w_{0i}}{\sum_k^3 w_{0k}} \quad (6)$$

Note: The parameters of the three border cells are all the same; thus, this is an unbiased network.

## Parameters and Initial Values

Let  $A_0 = 1, A = 5, B = C = D = 1, \Gamma = 0.2$ , and  $\tau = 0.05$ . Assume  $x_0(0) = 0, x_i(0) = (0.6, 0.1, 0.3)$ , and  $w_{0i}(0) = (0.7, 0.2, 0.1)$ . Assume that for the  $t \geq 0$ , the stimulus pattern  $I_i = (0.1, 0.7, 0.2)$  is presented to the border cells, and for  $t \geq 2$  a conditioning signal  $I_0 = 1$  is received by the source cell.

## Analysis (40 points)

Analytically determine the values of pattern variables  $X_i$  and  $W_{0i}$  as  $t \rightarrow \infty$ . Hints:

1. The first equation is uncoupled from the others, and thus it has a simple solution as  $t \rightarrow \infty$  since  $I_0$  is constant for  $t \geq 2$ .
2. After  $x_0$  is determined,  $S_0$  is easily determined
3. After  $S_0(t) = [x_0(t - \tau) - \Gamma]_+$  becomes positive, equations for  $x_i$  and  $w_{0i}$  can be written as a nonhomogenous autonomous system of linear ODEs with all eigenvalues having negative real parts.

Compare your result with results that follow from Outstar Learning Theorem.

## Computational Work

### Part 1 (30 points)

Numerically integrate the outstar equations from  $t = 0$  to  $t = 10$ . For each  $i$  plot

- the original variables  $I_i, x_i, w_{0i}$
- the pattern variables  $\Theta_i, X_i, W_{0i}$

through time.

Compare activations and weights (original and normalized) to the corresponding inputs at  $t = 10$ .

Compare your numerical values with analytical values from the previous section

### Part 2 (30 points)

Change the parameter  $A$  from 5 to 0.5, and think about what effect this may have on the coupled system. Write down your conclusions. Then rerun the simulation and make a set of plots similar to those in previous section.

Compare your results to those of previous sections of the assignment. Be sure to examine both the original and the normalized (pattern) variables before summarizing your results! Discuss your results in terms of the conditions on the Outstar Learning Theorem.